

Exercise 23

Find the derivative of the function using the definition of derivative. State the domain of the function and the domain of its derivative.

$$f(t) = 2.5t^2 + 6t$$

Solution

Calculate the derivative of $f(t)$ using the definition.

$$\begin{aligned} f'(t) &= \lim_{h \rightarrow 0} \frac{f(t+h) - f(t)}{h} \\ &= \lim_{h \rightarrow 0} \frac{[2.5(t+h)^2 + 6(t+h)] - (2.5t^2 + 6t)}{h} \\ &= \lim_{h \rightarrow 0} \frac{[2.5(t^2 + 2th + h^2) + 6t + 6h] - 2.5t^2 - 6t}{h} \\ &= \lim_{h \rightarrow 0} \frac{(2.5t^2 + 5th + 2.5h^2 + 6t + 6h) - 2.5t^2 - 6t}{h} \\ &= \lim_{h \rightarrow 0} \frac{5th + 2.5h^2 + 6h}{h} \\ &= \lim_{h \rightarrow 0} (5t + 2.5h + 6) \\ &= 5t + 6 \end{aligned}$$

The domain of $f(t)$ is $\{t \mid -\infty < t < \infty\}$, and the domain of $f'(t)$ is $\{t \mid -\infty < t < \infty\}$. $f(t)$ and $f'(t)$ are polynomials, so any number can be plugged into them.